# Modified Large Distance Newton Potential on a Gauss-Bonnet Brane World

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Gravity on a brane world with higher order curvature terms and a conformally coupled bulk scalar field is investigated. Solutions with non-standard large distance gravity are described. It is not necessary to include a scalar field potential in order to obtain the solutions. The resulting Newton potential is qualitatively similar to that of the Dvali-Gabadadze-Porrati (DGP) model. For suitable parameter choices the model is ghost free. Like many other brane gravity models with modified large distance Newton potentials, the short distance gravity is scalar-tensor. The scalar couples with gravitational strength, and so the model is not compatible with observation.

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## I. INTRODUCTION

In the brane world scenario our four dimensional universe is a brane embedded in a higher dimensional bulk space. There are many versions of this scenario, but in this paper we will be interested in models with a single brane in an infinite, five-dimensional bulk space. In the Randall-Sundrum (RS) model, the warping of the bulk traps a graviton zero mode on the brane [1]. This allows standard four dimensional gravity to be obtained there (at least for larger distance scales). Brane models which do not give conventional gravity at larger distances have also been proposed [2, 3, 4, 5, 6]. In this article we will be interested in a brane model which does not have a massless graviton bound state. The effective four-dimensional gravity is obtained by a different method. Such models have a non-standard large distance Newton potential. There are also corrections to it at shorter distances, which are of interest in an astrophysical context.

One popular model with modified large distance gravity is the Dvali-Gabadadze-Porrati (DGP) model [2]. It has an unwarped bulk, so that five dimensional gravity is obtained at large distances. Four dimensional gravity is obtained at short distances due an induced curvature term on the brane.

We will consider a brane model which has a conformally coupled scalar field and whose action includes the quadratic order Gauss-Bonnet curvature term. It is just as natural to include the Gauss-Bonnet term in the action as the usual Einstein-Hilbert term, since the resulting gravitational field equations are as well behaved as the Einstein equations [7]. We will also include second order kinetic terms for the scalar field. The literature contains many papers on linearised gravity in Gauss-Bonnet brane models [8, 9, 10]. We will review the relevant points in section II.

The boundary action associated with the Gauss-Bonnet term includes the induced Einstein tensor [11]. For a warped bulk spacetime, this will give a similar contribution to the effective four-dimensional theory as the induced gravity term in the DGP model. This suggests that solutions can be found with a similar behaviour to the DGP model. We will show in section III that, with the help of the scalar field, this is indeed the case. Our solutions do not appear to have the strong interaction problems of DGP model [12, 13].

Closer examination of the solutions reveals that they have much in common with quasilocalised brane gravity models [3, 4]. In these models the bulk is warped, but not sufficiently for there to be a bound graviton zero mode. Four dimensional gravity is instead obtained from the massive graviton states. Previous quasilocalised models have suffered from problems with ghosts [14]. For suitable parameter choices these can be avoided for our solutions.

Unfortunately the bulk gravitons in our model have extra an degree of freedom. The resulting four dimensional theory contradicts solar system observations, as it does not give the correct predictions for light bending. This problem is discussed in section IV.

In section V we consider the implications of our analysis for a model with only second order gravity terms in its action, and show that (for the class of solutions we are considering) all solutions are unstable.

## II. EFFECTIVE FOUR DIMENSIONAL GRAVITY

We will consider a brane model with the gravitational action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,e^{-2\Phi}\left\{a_1 R - 4a_2(\nabla\Phi)^2 + \mathcal{L}_2 - 2\Lambda\right\} - \frac{1}{\kappa^2}\int d^4x\sqrt{-h}\,e^{-2\Phi}\left\{2a_1 K + \mathcal{L}_2^{(b)} + T\right\} \qquad (1)$$

where $\Lambda$ and $T$ are respectively the bulk cosmological constant and brane tension, and $\mathcal{L}_2$ and $\mathcal{L}_2^{(b)}$ give the second order curvature contributions. We are treating the brane as a boundary of the bulk space, and so have included the Gibbons-Hawking boundary term [15]. $h_{ab} = g_{ab} - n_a n_b$ is the induced metric on the brane and $K_{ab} = h^c{}_a\nabla_c n_b$ is the extrinsic curvature.

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The quadratic order part of the action is

$$\mathcal{L}_2 = c_1\left(R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd}\right) - 16c_2 G_{ab}\nabla^a\Phi\nabla^b\Phi + 16c_3(\nabla\Phi)^2\nabla^2\Phi - 16c_4(\nabla\Phi)^4 \tag{2}$$

$$\mathcal{L}_2^{(b)} = \frac{4}{3}c_1\big(3KK_{ab}K^{ab} - 2K^a{}_bK^{bc}K_{ac} - K^3 - 6G^{(4)}_{ab}K^{ab}\big) - 16c_2(K_{ab} - Kh_{ab})D^a\Phi D^b\Phi + \frac{16}{3}c_3\left((n^a\partial_a\Phi)^3 + 3n^a\partial_a\Phi(D\Phi)^2\right) \ . \tag{3}$$

Again, appropriate boundary terms have been included [10, 11, 16]. The full bulk field equations and brane junction conditions obtained from the action (1) are given in ref. [10]. Note that $G_{ab}$ denotes the Einstein tensor, and not the induced metric.

We will be interested in perturbations of a RS-like background, with $ds^2 = e^{2A(z)}(dx_4^2 + dz^2)$, $\Phi'/A' = u =$ constant and $A(z) = -\ln(1 + |z|/\ell)$. We take $u$ to be constant for simplicity. Furthermore, the form of the resulting solutions is similar to the RS and DGP brane worlds, allowing easy comparison of the different models. The brane is at $z = 0$ and the bulk is taken to be $Z_2$ symmetric. We take $u$ and $\ell$ to be positive, as we are interested in RS-like solutions without bulk singularities. The relations between the different parameters are listed in the appendix.

We consider the perturbed metric

$$ds^2 = e^{2A}\left[(\eta_{\mu\nu} + \gamma_{\mu\nu})dx^\mu dx^\nu + dz^2\right] - \ell e^A dz(dx^\mu\partial_\mu + dz\partial_z)\left(N_1\psi + \frac{2}{u}\varphi\right) \tag{4}$$

and take $\Phi = \phi_0 + uA + \varphi$. The metric perturbation is expanded as

$$\gamma_{\mu\nu} = \bar\gamma_{\mu\nu} + 2(\zeta + \psi)\eta_{\mu\nu} - 2N_2\frac{\partial_\mu\partial_\nu}{\Box_4}\psi \tag{5}$$

where $\partial^\mu\bar\gamma_{\mu\nu} = 0$ and $\eta^{\mu\nu}\bar\gamma_{\mu\nu} = 0$. The fields $\psi$ and $\zeta$ are linear combinations of $\varphi$ and $\gamma = \eta^{\mu\nu}\gamma_{\mu\nu}$. We work in a gauge in which the brane does not bend, even when matter is added to it (see e.g. ref [10] for more details). This is achieved by suitable choices of $\zeta$, $\psi$ and the coefficients $N_1$, $N_2$ (see appendix).

The linearised bulk field equations give

$$\mu_\gamma\left(\partial_z^2 - (3-2u)\ell^{-1}e^A\partial_z + f_\gamma^2\Box_4\right)\bar\gamma_{\mu\nu} = 0 \tag{6}$$

where

$$\mu_\gamma = a_1 - 4\ell^{-2}(c_1(1-4u) + 2c_2u^2) \tag{7}$$

$$f_\gamma^2 = 1 + 8u\frac{c_1(1+2u) + 2c_2u}{\mu_\gamma\ell^2} \ . \tag{8}$$

If either of $\mu_\gamma$ or $f_\gamma^2$ are negative, the model will have graviton ghosts. This requirement restricts allowable ranges of the parameters $a_i$ and $c_i$ in the action. The graviton wave equation (6) is solved (for spacelike momenta $p$) by

$$\bar\gamma_{\mu\nu} \propto e^{-(2-u)A}K_{2-u}\left(f_\gamma p\ell e^{-A}\right) \ . \tag{9}$$

where $K_{2-u}$ is the order $(2-u)$ modified Bessel function of the second kind. The effective gravitational law on the brane is given by the junction conditions. We find

$$2\mu_\gamma\partial_z\bar\gamma_{\mu\nu} + m_\gamma^2\Box\bar\gamma_{\mu\nu} = -2\kappa_*^2\left\{S_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box_4}\right)S\right\} \tag{10}$$

where $S_{\mu\nu}$ is the perturbation of the energy momentum tensor on the brane and $\kappa_* = \kappa e^{\phi_0}$. The parameter $m_\gamma^2 = 8c_1(1-2u)/\ell$ must be positive, or the theory will have tachyonic graviton modes [9, 10].

The behaviour of the scalar field $\psi$ is qualitatively similar. Its bulk field equation is the same as eq. (6), but with $\mu_\gamma$ and $f_\gamma^2$ replaced by $\mu_\psi$ and $f_\psi^2$. Its junction condition is

$$2\mu_\psi\partial_z\psi + m_\psi^2\Box_4\psi = -\kappa_*^2 S \ . \tag{11}$$

Again ghosts and tachyons are present for some parameter ranges.

The remaining degree of freedom, $\zeta$, is pure gauge in the bulk, but its behaviour on the brane is fixed by the junction conditions

$$m_\zeta^2\Box_4\zeta = -\kappa_*^2 S \ . \tag{12}$$

If we require that $\zeta$ is not a ghost, $m_\zeta^2$ must be positive.

Requiring that the model is completely free from ghosts and tachyons will rule out large regions of parameter space, although solutions which do not suffer from either of these types of instabilities do exist. An example is given at the end of the next section.

## III. MODIFIED NEWTON POTENTIAL

By substituting the solution (9) into the junction conditions we can obtain the linearised effective four dimensional gravity. If we then consider a perturbation corresponding to a point mass at $r = 0$ on the brane, we can also determine the effective Newton potential $V_N$. The motion of a test particle confined to the brane is given by $d^2x^i/dt^2 \approx -{}^{(4)}\Gamma^i_{00}$, and from this we obtain $V_N = (1/2)\gamma_{00}$.

If $0 \le u < 1$ then for small $p$ (which corresponds to large distance scales) a series expansion of eq. (9) gives

$$\partial_z\bar\gamma_{\mu\nu} \approx -\frac{p^2 f_\gamma^2\ell}{2(1-u)}\bar\gamma_{\mu\nu} \ . \tag{13}$$

Thus the junction condition (10) implies $\bar\gamma_{\mu\nu} \propto \{S_{\mu\nu} - \cdots\}/p^2$. Fourier transforming this when $S_{\mu\nu}$ corresponds

to a point mass [so $S_{00} \propto \delta(p^0)$], we find it gives a standard $1/r$ contribution to Newton's law. This is qualitatively identical to what happens in the RS model (which can be obtained from the above case by taking $u = 0$). Alternatively, if we look at the graviton spectrum for this model, we will see that it has a zero mode which is localised on the brane. This mode gives the dominant contribution to the large distance gravity of this type of solution.

If $1 < u < 2$ then the expression (13) is no longer valid, and instead we have

$$\partial_z \bar{\gamma}_{\mu\nu} \approx -\frac{2\Gamma(u-1)}{\ell\Gamma(2-u)} \left(\frac{p\ell f_\gamma}{2}\right)^{4-2u} \bar{\gamma}_{\mu\nu} \qquad (14)$$

at large distances. Substituting eq. (14) into the junction condition (10), we again obtain $\bar{\gamma}_{\mu\nu}$ in terms of the energy momentum perturbation $S_{\mu\nu}$. We now find that it gives a non-standard $1/r^{2u-1}$ contribution to the large distance Newton potential. This is the same as would be obtained for a $2(u+1)$-dimensional theory. A special case of this is $u = 3/2$, for which $\partial_z \bar{\gamma}_{\mu\nu} = -f_\gamma p \bar{\gamma}_{\mu\nu}$ at all scales. The large distance contribution to the Newton potential is then $1/r^2$, which is the same as is obtained in the DGP model. The scalar mode, $\psi$, gives similar non-standard contributions to the large distance Newton's law.

We now have only massive graviton bound states and no localised zero mode. However we can still obtain four-dimensional gravity at short distances, since in this case the $m_\gamma^2 \Box_4 \bar{\gamma}_{\mu\nu}$ term dominates the junction condition (10), and (to leading order) a standard $1/r$ Newton potential is obtained.

Putting all the junction conditions together, we obtain the following expression for the induced metric perturbation

$$\gamma_{\mu\nu}(p) = 2\kappa_*^2 \Bigg( D_\gamma(p) \left\{ S_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) S \right\} + D_\psi(p) \left\{ \eta_{\mu\nu} - N_2 \frac{p_\mu p_\nu}{p^2} \right\} S + \eta_{\mu\nu} \frac{S}{m_\zeta^2 p^2} \Bigg) \qquad (15)$$

where

$$D_i(p) = \left\{ m_i^2 p^2 + 2\mu_i f_i p \frac{K_{u-1}(f_i \ell p)}{K_{2-u}(f_i \ell p)} \right\}^{-1} . \qquad (16)$$

The three contributions to eq. (15) correspond respectively to the bulk graviton modes, the bulk scalar, and the 'brane-bending' mode. The Newton potential, $V_N$, and the effective four-dimensional graviton propagator can be extracted from the above expression (15).

Defining $r_i = m_i^2/(2\mu_i f_i)$, we find that for large momentum $D_i^{-1}(p) = m_i^2[p^2 + p/r_i + O(1)]$. Using this asymptotic behaviour of $D_i(p)$, and taking the perturbation $S_{\mu\nu}$ to be a point mass, we can determine $\gamma_{00}$. By fourier transforming this we can obtain the approximate short distance Newton potential

$$V_N = -\frac{\kappa_*^2}{4\pi r} \left( \frac{2}{3m_\gamma^2} + \frac{1}{m_\psi^2} + \frac{1}{m_\zeta^2} \right) - \frac{\kappa_*^2}{2\pi^2} \left\{ \frac{2}{3r_\gamma} \ln\left(\frac{r}{r_\gamma}\right) + \frac{1}{r_\psi} \ln\left(\frac{r}{r_\psi}\right) \right\} + O(1) . \qquad (17)$$

which will be valid when $r \ll r_\gamma, r_\psi, \ell f_\gamma, \ell f_\psi$. The logarithmic corrections are similar to those that appear in the DGP model. In fact, they will be present even for solutions with localised ($0 \le u < 1$) gravity. Similar corrections also occur in warped space versions of the DGP model [17].

In a similar way we can obtain the large distance effective gravity by using a power series expansion of $D_i(p)$ in the expression for $\gamma$ (15). If $1 < u < 2$, then for large distances ($r \gg r_\gamma, r_\psi, \ell f_\gamma, \ell f_\psi$)

$$V_N = -\frac{\kappa_*^2}{4\pi m_\zeta^2 r} - \frac{\kappa_*^2 \Gamma(u-1/2)}{4\pi^{3/2}\Gamma(u-1)} \times \left( \frac{2}{3\mu_\gamma f_\gamma^{4-2u}} + \frac{1}{\mu_\psi f_\psi^{4-2u}} \right) \frac{\ell^{2u-3}}{r^{2u-1}} \qquad (18)$$

to leading order. Note that the power series expansion of $K_{u-1}$ used to derive eq. (18) is only valid for $u > 1$, and so putting $u = 0$ in the above expression does not give the correct Newton potential for the RS model.

If $m_\zeta^2$ is large, the resulting large and short distance gravity resembles the DGP model, especially for the special case of $u = 3/2$. We then have $1/r^2$ contributions to the large distance Newton's law, as would normally occur in five-dimensional gravity. However, even if $m_\zeta^2$ is very large, the first term in the above expression (18) will eventually dominate. We will therefore have a $1/r$ Newton potential at very large distances, in contrast to the DGP model. At intermediate scales, we will have a combination of logarithmic and $1/r^{2u-1}$ corrections.

One problem with previous quasilocalised brane gravity models is that they have ghosts [14]. For a wide range of parameter choices, our solutions have the same problem, although there are exceptions. For example suppose $u$ is fairly close to 1 ($1 < u \lesssim 1.07$) and the parameters in the action (1) are $c_2 = (7-u)c_1/4$, $c_3 = (13 - 73u + 126u^2 - 18u^3)c_1/(16u^3)$, $c_4 = (13 - 64u + 60u^2 + 41u^3 - 14u^4)c_1/(8u^4)$, $a_1 = (u-1)(8u-7)c_1/\ell^2$ and $a_2 = 0$. If $c_1$ is negative, then $m_i^2, \mu_i > 0$ and the solution is free from ghosts and tachyons. To leading order in $u-1$ we find $\mu_\gamma \ell \sim m_\gamma^2 \sim m_\zeta^2 \sim -(u-1)c_1/\ell$, $f_\psi^2 \sim (1-u)^2$, $f_\gamma^2 \sim 1$, $m_\psi^2 \sim -c_1/\ell$ and $\mu_\psi \sim -c_1/\ell^2/(u-1)$. Hence we have shown (by construction) that ghost free solutions with a non-standard large distance Newton's law can be found. The above solution also shows that a modified Newton's law which is very close to the usual $1/r$ four dimensional one can be found.

Although the DGP model does not have ghosts, it does have strong interactions at short distances, raising doubts about the model's predictivity [12, 13]. It was shown in ref. [12] that the strong interaction scale comes from the small kinetic term for the brane bending mode. In our model this is $m_\zeta^2$, which is not generally small, suggesting that there need not be a corresponding strong interaction problem for our model.

## IV. RELATIVISTIC EFFECTS

While our model does give the correct Newton potential at short distances, this is not sufficient for it to be compatible with gravity measurements of the solar system. At short distances ($1/p \ll \ell f_{\gamma,\psi}, r_{\gamma,\psi}$) the leading order behaviour of the perturbation (15) is

$$\gamma_{\mu\nu}(p) \approx \frac{2}{M_{\rm Pl}^2 p^2}\left\{S_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}S\right\} + \frac{1}{M_{\rm S}^2 p^2}\ \eta_{\mu\nu}S\ . \quad (19)$$

We have omitted the $p_\mu p_\nu/p^2$ dependent terms for simplicity. This describes scalar-tensor gravity, with Planck mass $M_{\rm Pl} = m_\gamma/\kappa_*$ and scalar mass $M_{\rm S}$ given by

$$\frac{1}{M_{\rm S}^2} = 2\kappa_*^2\left(\frac{1}{6m_\gamma^2} + \frac{1}{m_\psi^2} + \frac{1}{m_\zeta^2}\right)\ . \quad (20)$$

To avoid conflict with solar system gravity tests, we need $M_{\rm S} \gg M_{\rm Pl}$ [18, 19]. This is not possible if $m_\zeta^2$ and $m_\psi^2$ are both positive.

The problem arises because the contribution from the $\bar{\gamma}_{\mu\nu}$ modes [the first term of eq. (15)] has the wrong tensor structure. The presence of the factor 1/3, as opposed to 1/2, indicates that there is an extra degree of freedom, the graviscalar, which couples to the energy-momentum tensor. A similar situation occurs in massive four-dimensional gravity. The effects of this extra graviscalar are not compatible with observations [18]. Our model also has the additional $\psi$ and $\zeta$ scalar modes. These will make things even worse, although their effects could be suppressed by taking $m_\zeta^2, m_\psi^2 \gg m_\gamma^2$. A similar problem occurred with quasilocalised gravity models [14]. This was originally thought to be a problem with the DGP model, although the situation is actually more complicated due to the model's strong interactions. In general, it seems that our model does not have this feature, suggesting its predictions are more reliable. However since these predictions appear to be incompatible with observations, this is not very useful.

To obtain an effective four dimensional theory with correct relativistic behaviour, the contribution of the extra graviscalar must be cancelled, either with the brane-bending field, $\zeta$, or the bulk scalar, $\psi$. This is not possible unless we have either $m_\zeta^2$ (or $m_\psi^2$) is negative, as was the case for quasilocalised gravity models. Looking at the linearised action for these scalar fields

$$-\frac{1}{2\kappa_*^2}\int d^5x\, e^{(3-2u)A}\mu_\psi\left\{(\partial_z\psi)^2 + f_\psi^2(\partial_\nu\psi)^2\right\}$$
$$-\frac{1}{2\kappa_*^2}\int d^4x\left\{m_\psi^2(\partial_\nu\psi)^2 + m_\zeta^2(\partial_\nu\zeta)^2\right\} \quad (21)$$

we see that if $m_\zeta^2 < 0$ the theory will have a ghost. If $m_\psi^2 < 0$ then either $\psi$ is a ghost (if $\mu_\psi < 0$), or it has a tachyon mode [since eq. (11) has solutions with spacelike momenta even in the vacuum].

In the RS model $m_\zeta^2$ is negative, but this is not a problem. This model has two massless graviton zero modes, and an unphysical graviscalar zero mode. The brane-bending ghost zero mode cancels the graviscalar (a similar idea is used in the quantisation of QED). This does not work for quasilocalised gravity models since there are no massless graviscalar or scalar states for the ghost to cancel with [14]. Furthermore, the resulting Newton potential is repulsive at large $r$.

For our model it is possible to cancel the contribution of the massive graviscalar modes by making $\psi$ a ghost. In general this is likely to be a physical ghost, in contrast to $\zeta$ in the RS model. However if $\psi$ and $\bar{\gamma}_{\mu\nu}$ have exactly the same spectrum, this may not be the case. For this we need $f_\psi = f_\gamma$ and $\mu_\psi/m_\psi^2 = \mu_\gamma/m_\gamma^2$. To get $M_{\rm S} \gg M_{\rm Pl}$ we also need $m_\psi^2 \approx -m_\gamma^2/6$ and $m_\zeta^2 \gg m_\gamma^2$. The resulting effective four dimensional gravity would be compatible with observations, at least to linear order. To know if $\psi$ is a physical ghost or not, we would have to go to higher order in perturbation theory, and calculate interaction terms.

In fact, having $m_\psi^2$ negative may not be as bad as it seems. Normally such a ghost field would imply that the energy of the theory is unbounded from below. However, the higher order terms in the action mean that terms such as $(\partial\psi)^4$ will also be present. If the effective action contains a term such as $-C_1(\partial_\mu\psi\partial_\nu\psi - C_2\eta_{\mu\nu})^2$, with $C_1, C_2 > 0$, then $m_\psi^2$ is negative, but the theory's energy is still bounded from below.

Throughout this article we have assumed that a linearised analysis is sufficient to determine the effective gravity in our model. If any of $m_i$ are zero or very small, this will not be the case and the expression for $M_{\rm S}$ (20) will not be valid. It is possible that by going to quadratic order we will obtain brane gravity that more closely resembles general relativity at shorter distances. A similar situation arises in the model considered in ref. [20], where conventional gravity was obtained at intermediate length scales when the analysis was extended beyond linear order.

Even if there is no viable solution to the problem of the extra degrees of freedom in our modified gravity model, the above analysis can still be used to impose constraints on the parameters of the underlying theory. For the action (1) there are still solutions with $u < 1$ and RS-style localised gravity. This is in contrast to the corresponding first order gravity theories, which have no solutions with localised gravity.

## V. PURE SECOND ORDER GRAVITY

If we remove either the first or second order terms from the action, then the algebraic expressions for the various coefficients in the previous sections simplify considerably. If $a_i = 0$ then

$$f_\gamma^2 = -1 + \frac{\ell m_\gamma^2}{\mu_\gamma}(u-1)\ . \quad (22)$$

To avoid graviton tachyons and ghosts we need $\mu_\gamma$, $m_\gamma^2 = 8c_1(1-2u)/\ell$ and $f_\gamma^2$ to be positive. This is not possible unless $u > 1$ and $c_1, c_2 < 0$. Thus we cannot get stable solutions with localised gravity unless both first and second order terms are included in the action.

Furthermore

$$\mu_\psi f_\psi^2 = -6\ell\mu_\gamma T \frac{T\ell f_\gamma^2 + 6(u-1)\mu_\gamma}{(\ell T - 6\mu_\gamma)^2} \; . \tag{23}$$

To have a ghost free theory we require $m_\zeta^2 = \ell^2 T$, $f_\gamma^2$, $\mu_\gamma$ and the above expression to all be positive. Since $u > 1$, this is not possible for any choice of parameters and so the theory always has ghosts.

As we discussed in the previous section, if $\psi$ is a ghost, it is conceivable that it could cancel with the graviscalar. For this to work we would need $f_\psi = f_\gamma$ and $\mu_\psi/m_\psi^2 = \mu_\gamma/m_\gamma^2$. If $\mu_\gamma > 0$ this occurs when $c_3 = (2c_2/c_1 - 1)c_2$ and $c_4 = c_3 c_2/c_1$. However this is not acceptable since in this case $m_\zeta^2 = 2\ell\mu_\gamma(3-2u) < 0$, and so we have a massless, physical ghost.

Hence we can rule out all pure second order gravity solutions, since they all have ghosts or tachyons. Pure first order gravity (with a scalar field) has similar problems.

## VI. SUMMARY

We have investigated the effective four-dimensional gravity of a brane world scenario with a conformally coupled scalar field and quadratic order curvature terms. As well as localised gravity solutions which give a conventional large distance Newton potential, there also exist solutions with modified large distance gravity.

These solutions have features of both the DGP and quasilocalised gravity scenarios. The bulk space is warped, but we can still obtain a DGP-like Newton's law, and do not appear to have the strong interaction problems of that model. The theory's higher curvature terms allow a short distance Newton's law with $1/r$ behaviour to be obtained. It has logarithmic corrections like those in the DGP model. The large distance Newton's law includes $1/r^{2u-1}$ terms, with $1 < u < 2$. For $u = 3/2$ this resembles the large distance behaviour of the DGP model.

Examination of the tensor structure of the graviton propagator reveals that at short distances the effective gravity of the solution is scalar-tensor, and that the scalar and gravitational couplings are of similar strength. This is incompatible with solar system gravity experiments. It may be possible to fix the problem by making the bulk scalar field a ghost, although the resulting theory is unlikely to be consistent.

For the special case of only second order curvature terms (and corresponding scalar kinetic terms), all solutions have either physical ghosts or tachyons. We see that when a scalar field is included in the model, solutions with well behaved effective gravity can only be found if the bulk action includes both first and second order curvature terms.

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## APPENDIX A: PARAMETERS

The bulk cosmological constant and brane tension are respectively $\Lambda = \Lambda^{(1)} + \Lambda^{(2)}$ and $T = T^{(1)} + T^{(2)}$ where

$$\Lambda^{(1)} = \frac{2}{\ell^2}(a_1(4u-3) + a_2 u^2) \tag{A1}$$

$$\Lambda^{(2)} = \frac{4}{\ell^4}(3c_1(1-8u) + 36c_2u^2 - 4c_3(4+u)u^3 + 6c_4u^4) \tag{A2}$$

$$T^{(1)} = \frac{2}{\ell}(3-2u)a_1 \tag{A3}$$

$$T^{(2)} = -\frac{8}{3\ell^3}(3c_1(1-6u) + 18c_2u^2 - 4c_3u^3) \; . \tag{A4}$$

We will find it useful to define $\Lambda_+ = 2\Lambda^{(1)} + 4\Lambda^{(2)}$ and $T_\pm = \pm T^{(1)} + 3T^{(2)}$.

The parameter $u = \Phi'/A'$ satisfies the equation

$$\begin{aligned} a_1 + 2(a_1+a_2)u - 4\ell^{-2}[3c_1 + 6(c_1 - 2c_2)u \\ + 2(4c_3 - 3c_2)u^2 + 4(c_3 - c_4)u^3] = 0 \; . \end{aligned} \tag{A5}$$

In the expressions for the metric perturbation (4,5), the coefficients are

$$N_1 = 12\frac{\mu_\gamma(T_+\ell + 4a_1[2-3u])}{T_+\ell(T_+\ell + 8a_1u - 6\mu_\gamma)} \tag{A6}$$

$$N_2 = \left(\frac{\Lambda_+}{T_+} + \frac{T_+}{3\mu_\gamma}\right) N_1\ell \tag{A7}$$

and the scalar fields are

$$\psi = \frac{T_+}{2N_1\ell\Lambda_+u}(8\varphi - u\gamma) \tag{A8}$$

$$\zeta = \frac{\varphi}{u} + \frac{6(\mu_\gamma - 2a_1)}{T_+\ell + 8a_1u - 6\mu_\gamma}\psi \; . \tag{A9}$$

The coefficients in the wave equations for these fields are

$$\mu_\psi = \frac{N_1^2\ell^2}{4}\left(\Lambda_+ + \frac{T_+^2}{3\mu_\gamma}\right) \tag{A10}$$

$$f_\psi^2 = -\frac{N_1^2 T_+ \ell}{4\mu_\psi}\left(\frac{\ell T_+}{6\mu_\gamma} f_\gamma^2 + 1 - u\right) \tag{A11}$$

$$m_\psi^2 = -\frac{T_+ N_1^2 \ell^2}{4}\left(1 + \frac{T_+ m_\gamma^2}{6\mu_\gamma^2} + \frac{T_+(\mu_\gamma - 2a_1)^2}{T_- \mu_\gamma^2}\right) \tag{A12}$$

$$m_\zeta^2 = \ell^2 T_- \ . \tag{A13}$$


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